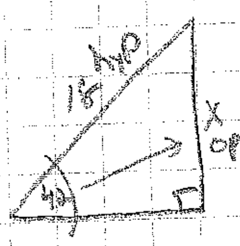
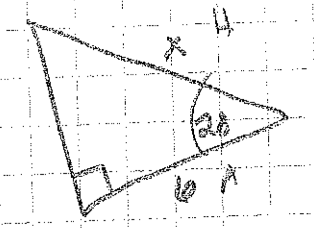
Unit 5 – Trig. Applications

Basic Right Δ Trig

Find x to the nearest hundredth:





$\sin\theta = \frac{O}{H}$

$\frac{\sin 42}{1} = \frac{x}{18}$

$x = 18 \cdot \sin 42$

$x = 12.04435081$

$\boxed{x = 12.04}$

$\cos\theta = \frac{A}{H}$

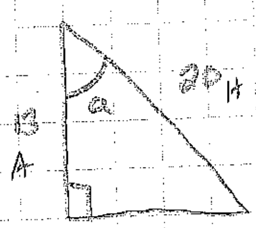
$\frac{\cos 28}{1} = \frac{6}{x}$

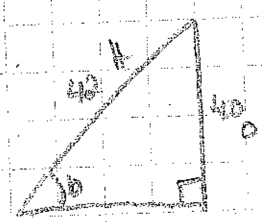
$\frac{\cos 28 \cdot x}{\cos 28} = \frac{6}{\cos 28}$

$x = 6.795420804$

$\boxed{x = 6.80}$

Find the value of the labeled angle to the nearest degree:





$\cos A = \frac{A}{H}$

$\cos A = \frac{13}{20}$

$A = \cos^{-1}(13/20)$

$A = 49.45839813$

$\boxed{A = 49^\circ}$

$\sin B = \frac{O}{H}$

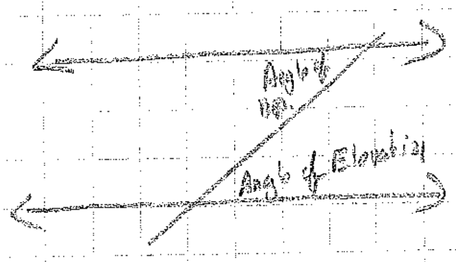
$\sin B = \frac{40}{42}$

$B = \sin^{-1}(40/42)$

$B = 72.24720984$

$\boxed{B = 72^\circ}$

Angles of Elevation & Depression

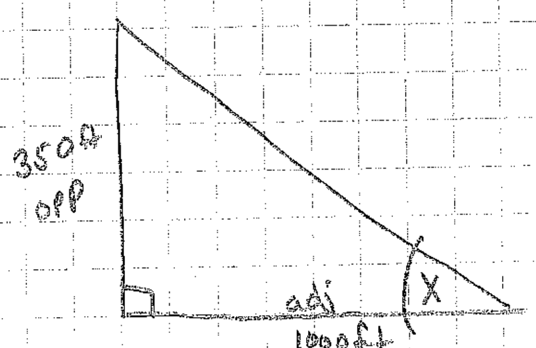


What do you notice about the angles of elevation and depression?

They are the same

Examples

1. A man standing on level ground is 1000 feet away from the base of a 350-foot-tall building. Find, to the *nearest degree*, the measure of the angle of elevation to the top of the building from the point on the ground where the man is standing.



$\tan\theta = \frac{O}{A}$

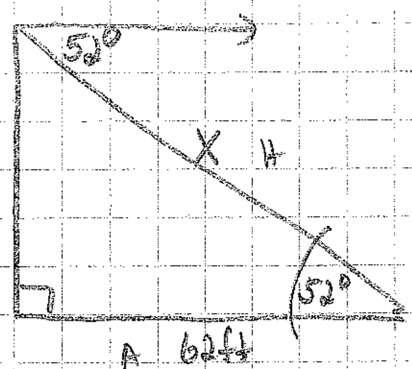
$\tan X = \frac{350}{1000}$

$X = \tan^{-1}(350/1000)$

$X = 19.29004622$

$\boxed{X = 19^\circ}$

2. A person measures the angle of depression from the top of a wall to a point on the ground. The point is located on level ground 62 feet from the base of the wall and the angle of depression is 52°. To the *nearest tenth*, how far is the person from the point on the ground?



$\cos \theta = \frac{A}{H}$

$\frac{\cos 52}{1} = \frac{62}{X}$

$\frac{\cos 52 \cdot X}{\cos 52} = \frac{62}{\cos 52}$

$X = 100.7046932$

$X = 100.7$ ft

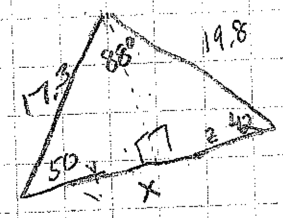
HW WS

Unit 5 Day #2 - Law of Sines

Objective: You will ① find the missing side or ∡ of a Δ using law of sines

How do we solve the following?

Find x:



$\frac{\cos 50}{1} = \frac{y}{17.3}$ $\frac{\cos 42}{1} = \frac{z}{19.8}$

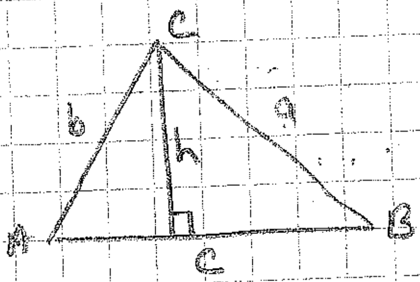
$y = 11.120\ldots$ $z = 14.714\ldots$

$x = 11.120\ldots + 14.714\ldots$

$= 25.8344$

$\boxed{= 25.8}$

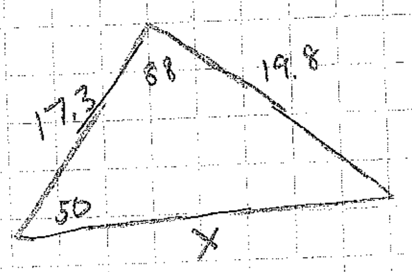
There is a formula to make it easier!



Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

* Used for working with 2 sides and 2 angles of any triangle.

So... Find x to the nearest hundredth.



$\frac{a}{\sin A} = \frac{b}{\sin B}$

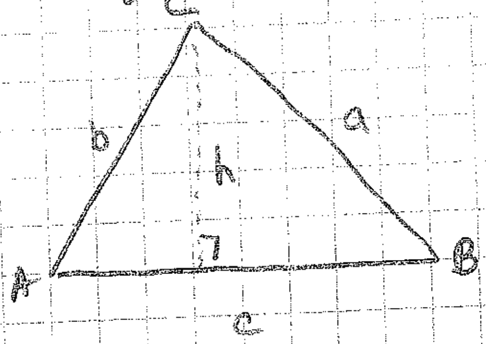
$\frac{x}{\sin 88} = \frac{19.8}{\sin 50}$

$\frac{\sin 50 \cdot x}{\sin 50} = \frac{19.8 \cdot \sin 88}{\sin 50}$

$x = 25.83131\ldots$

$\boxed{x = 25.8}$

Law of Sines:

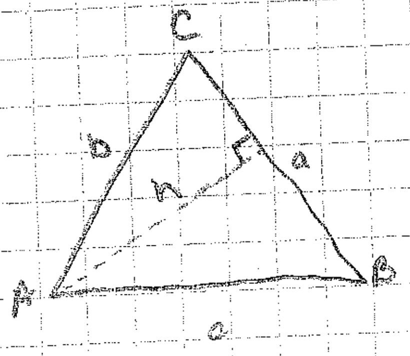


$\sin A = \frac{h}{b}$ $\qquad$ $\sin B = \frac{h}{a}$

$h = b \cdot \sin A$ $\qquad$ $h = a \cdot \sin B$

$$b \cdot \sin A = a \cdot \sin B$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$



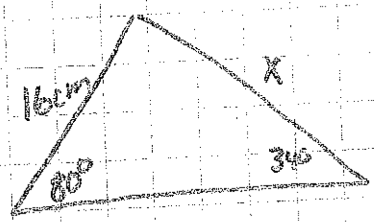
$\sin C = \frac{h}{b}$ $\qquad$ $\sin B = \frac{h}{c}$

$h = b \cdot \sin C$ $\qquad$ $h = c \cdot \sin B$

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

Examples:

1) Find x to the nearest tenth:



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

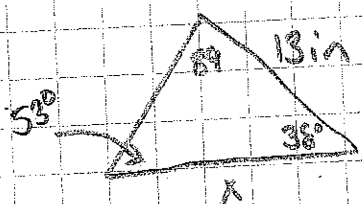
$$\frac{x}{\sin 80} = \frac{16}{\sin 34}$$

$$\frac{\sin 34 \cdot x}{\sin 34} = \frac{16 \cdot \sin 80}{\sin 34}$$

$x = 28.1779757$

$\boxed{x = 28.2 \text{ cm}}$

2) Find x to the nearest hundredth:



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

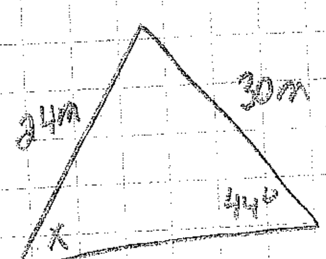
$$\frac{x}{\sin 89} = \frac{13}{\sin 53}$$

$$\frac{\sin 53 \cdot x}{\sin 53} = \frac{13 \cdot \sin 89}{\sin 53}$$

$x = 16.27528437$

$\boxed{x = 16.28}$

3) Find x to the nearest degree:



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{30}{\sin x} = \frac{24}{\sin 44}$$

$$\frac{24 \cdot \sin x}{24} = \frac{30 \cdot \sin 44}{24}$$

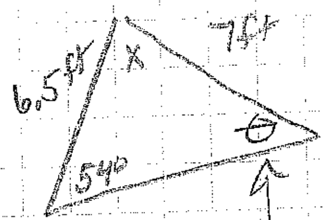
$\sin x = .8683229631$

$x = \sin^{-1}(.8683\ldots)$

$x = 60.26433799$

$\boxed{x = 60°}$

4) Find x to the nearest degree:



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{6.5}{\sin\theta} = \frac{7}{\sin 54}$$

$$\frac{7 \cdot \sin\theta}{7} = \frac{6.5 \cdot \sin 54}{7}$$

$\sin\theta = .7512300662$

$\theta = \sin^{-1}(.7512300662)$

$\theta = 48.697...$

$\theta = 49°$

$$\begin{array}{r} 54 \\ +49 \\ \hline 103° \end{array}$$

$x = 180 - 103$

$\boxed{x = 77°}$

<u>Hw:</u>

1) In $\triangle ABC$, $\angle A = 102$, $\angle B = 40$, $b = 8$
Find a to the nearest tenth.

2) In $\triangle COW$, $\angle C = 42°$, $\angle O = 71°$, $c = 11$
Find w to the nearest tenth.

3) In $\triangle BUG$, $\angle B = 38°$, $b = 17$, $g = 19$
Find $\angle G$ to the nearest degree.

4) In $\triangle XYZ$, $\angle Y = 73°$, $y = 23$, $z = 21$
Find $\angle X$ to the nearest degree.

Day #3 Lesson → See Worksheet → Applying <u>Law of Sines</u>

Unit 5 Day #4 → The Ambiguous Case (Law of Sines)

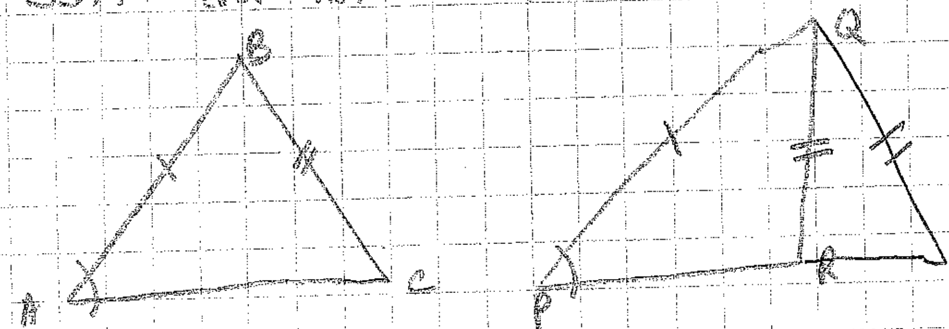
Objective - You will determine if a triangle has zero, one, or two solutions.

Recall: In Geometry there were 5 ways of proving triangles congruent. What are they?

SSS AAS
SAS HL
ASA

What did not work? ASS or SSA
(Donkey Theorem)

SSA did not work because....



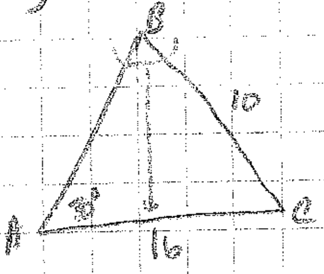
potentially 2 different triangles could be formed when given 1 angle measure and 2 side lengths (where the angle is NOT between the 2 sides).

The word ambiguous means open to two or more interpretations. When given 2 sides and 1 angle of a triangle, the Law of Sines could possibly give 0, 1, or 2 solutions.

Examples

Determine how many possible triangles can be formed when given the following:

1) In ΔABC, $a=10$, $b=16$, and $\angle A = 30°$



$$\frac{16}{\sin B} = \frac{10}{\sin 30}$$

$$\frac{10 \cdot \sin B}{10} = \frac{16 \cdot \sin 30}{10}$$

$\sin B = .8$

$B = \sin^{-1}(.8)$

$B = 53.13010235$

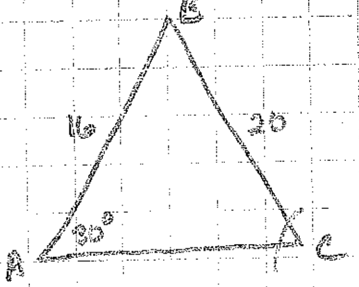
$\boxed{B = 53°}$ I

II: $180 - 53 = 127°$

	I	II
	A = 30°	A = 30°
	B = 53°	B = 127°
	C = 97°	C = 23°

$\therefore$ 2 Δ's

2) In ΔABC, $a=20$, $c=16$, and $\angle A = 30°$



$$\frac{16}{\sin C} = \frac{20}{\sin 30}$$

$$\frac{20 \cdot \sin C}{20} = \frac{16 \cdot \sin 30}{20}$$

$\sin C = .4$

$C = \sin^{-1}(.4)$

$C = 23.5781784$

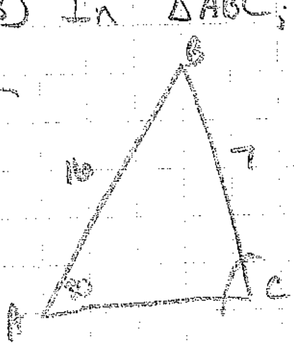
$\boxed{C = 24°}$ I

II: $180 - 24 = 156°$

	I	II
A	30°	30°
B	126°	X Not Possible
C	24°	156°
		186°

$\therefore$ 1 Triangle

3) In ΔABC; $a=7$, $c=16$, $\angle A = 30$



$$\frac{16}{\sin C} = \frac{7}{\sin 30}$$

$$\frac{7 \cdot \sin C}{7} = \frac{16 \cdot \sin 30}{7}$$

$\sin C = 1.142\ldots$

$C = \sin^{-1}(\text{Ans})$

$C =$ ERROR

$\therefore$ No Δ exists

HW: Determine the number of Δ's that can be formed

1) In ΔABC, $a = 12.5$, $b = 8.4$ $\angle A = 155°$

2) In ΔABC $a = 8$, $b = 14$ $\angle A = 20°$

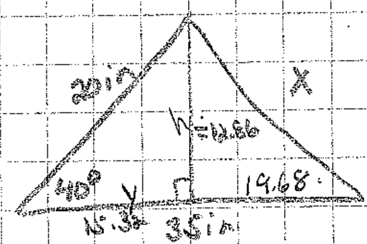
3) In ΔABC $a = 18$, $b = 20$ $\angle A = 65°$

Unit 5 Day #5 - The Law of Cosines

Objective - You will be able to solve triangles using the Law of Cosines.

How do we solve the following?

Find x to the nearest tenth:



$\frac{\sin 40}{1} = \frac{h}{20}$

$h = 20 \cdot \sin 40$

$h = 12.85575219$

$h = 12.86$

$\frac{\cos 40}{1} = \frac{y}{20}$

$y = 20 \cdot \cos 40$

$y = 15.3208886$

$y = 15.32$

$35 - 15.32$

$= 19.68$

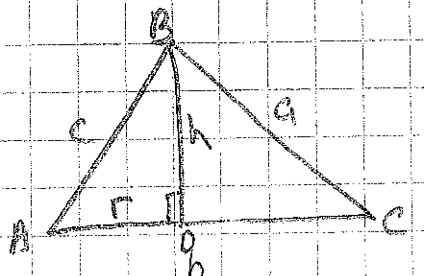
$12.86^2 + 19.68^2 = x^2$

$\sqrt{552.682} = \sqrt{x^2}$

$x = 23.50918969$

$\boxed{x = 23.5}$

There is a formula to make it easier!



$\sin A = \frac{h}{c} \Rightarrow h = c \cdot \sin A$

$\cos A = \frac{r}{c} \Rightarrow r = c \cdot \cos A$

Pyth Thm: $a^2 = h^2 + (b-r)^2$

$a^2 = (c \cdot \sin A)^2 + (b - c\cos A)^2$

$(b - c\cos A)$
$(b - c\cos A)$
$b^2 - bc\cos A - bc\cos A + c^2\cos^2 A$

$a^2 = c^2\sin^2 A + b^2 - bc\cos A - bc\cos A + c^2\cos^2 A$

$a^2 = c^2\sin^2 A + b^2 - 2bc\cos A + c^2\cos^2 A$

$a^2 = c^2\sin^2 A + c^2\cos^2 A - 2bc\cos A + b^2$

$a^2 = c^2(\sin^2 A + \cos^2 A) + b^2 - 2bc\cos A$

$a^2 = c^2(1) + b^2 - 2bc\cos A$

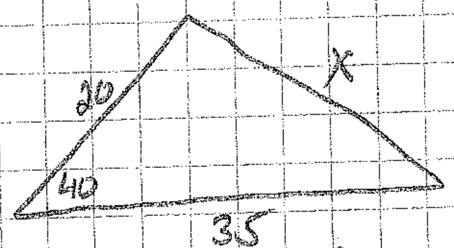
$$\boxed{a^2 = c^2 + b^2 - 2bc\cos A}$$

Law of Cosines

Used when working with 3 sides and 1 angle of any triangle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

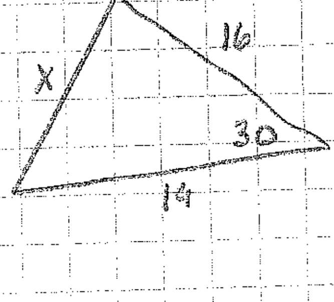
Find x to the nearest tenth:



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 20^2 + 35^2 - 2(20)(35)\cos 40$$
$$\sqrt{x^2} = \sqrt{1625 - 1400\cos 40}$$
$$x = 23.50612217$$
$$\boxed{x = 23.5}$$

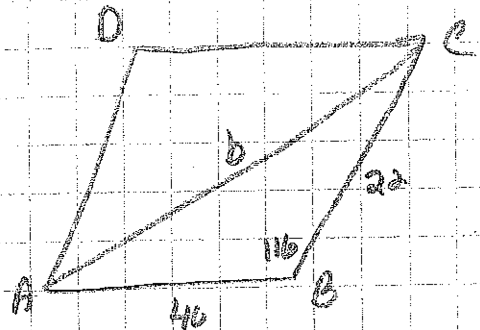
<u>Examples:</u>

1) Find x to the nearest tenth:



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 16^2 + 19^2 - 2(16)(19)\cos 30$$
$$\sqrt{x^2} = \sqrt{617 - 608\cos 30}$$
$$x = 9.510865076$$
$$\boxed{x = 9.5}$$

2) In a parallelogram, the adjacent sides measure 40cm and 22 cm. If the larger angle of the parallelogram measures 116, find the length of the larger diagonal to the nearest integer.



$b^2 = a^2 + c^2 - 2ac \cos B$

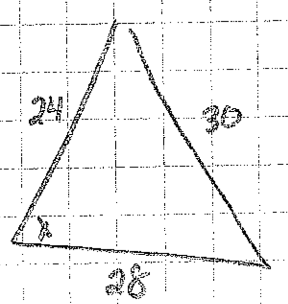
$b^2 = 22^2 + 40^2 - 2(22)(40) \cos 116$

$\sqrt{b^2} = \sqrt{2084 - 1760 \cos 116}$

$b = 53.4371894\overline{7}$

$\boxed{b = 53 \text{ cm}}$

3) Find x to the nearest degree:



$a^2 = b^2 + c^2 - 2bc \cos A$

$30^2 = 24^2 + 28^2 - 2(24)(28) \cos A$

$900 = 1360 - 1344 \cos A$

$\underline{-1360 \quad -1360}$

$\frac{-460}{-1344} = \frac{-1344 \cos A}{-1344}$

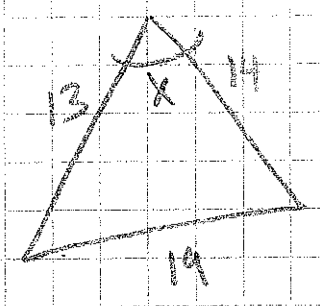
$\cos A = .3422619048$

$A = \cos^{-1}(.3422619048)$

$A = 69.98525841$

$\boxed{A = 70^\circ}$

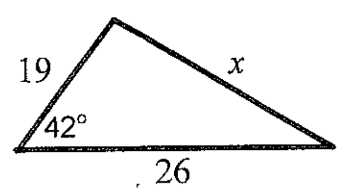
4) A triangle has side lengths of 13, 14, and 19. Find the largest angle of the Δ to the nearest tenth of a degree.



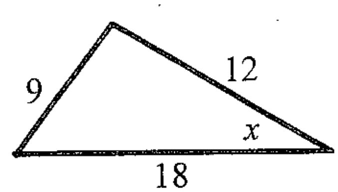
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$19^2 = 13^2 + 14^2 - 2(13)(14)\cos A$$
$$361 = 365 - 364\cos A$$
$$\underline{-365 \quad -365}$$
$$\frac{-4}{-364} = \frac{-364\cos A}{-364}$$
$$\cos A = .010989011$$
$$A = \cos^{-1}(.010989011)$$
$$A = 89.37036338$$
$$\boxed{A = 89.4^\circ}$$

Homework

1. Find x to the *nearest tenth:*



2. Find x to the *nearest degree:*



3. A triangular walking course has 2 sides of 230 feet and 360 feet, and the angle between those sides measures 38°. Find the length of the third side of the course to the *nearest foot.*

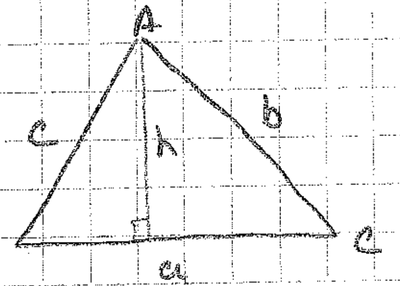
4. In a rhombus with a side of 24, the longer diagonal is 36. Find, to the *nearest degree*, the larger angle of the rhombus.

Unit 5 Day #6 - Area of a Triangle

<u>Objective</u> - You will find the area of a triangle.

Area of a Triangle

How do we find the area of a non-right triangle?



$A = \frac{1}{2} a \, h$ . . . Solve for h:

$c \cdot \sin B = \frac{h}{c} \cdot c$

$\boxed{A = \frac{1}{2} a \cdot c \sin B}$

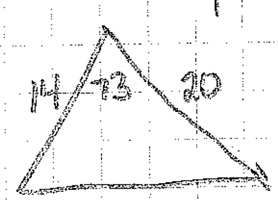
$\boxed{h = c \cdot \sin B}$

This formula can be used to find the area of a triangle when working with 2 sides and an included angle:

$$K = \frac{1}{2} a b \sin C$$

Examples:

1) Find the area to the nearest tenth of a square inch:



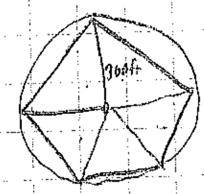
$K = \frac{1}{2} ab \sin C$

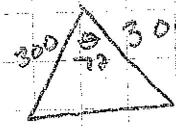
$K = \frac{1}{2}(14)(20) \sin 73$

$K = 133.8826658$

$\boxed{K = 133.9 \text{ in}^2}$

2) The center of the Pentagon in Washington DC is a courtyard in the shape of a regular pentagon. If the pentagon was inscribed in a circle, the radius would be 300 ft. Find the area of the courtyard to the nearest square foot.





$\theta = \frac{360}{5} = 72$

$K = \frac{1}{2} ab \sin C$

$K = \frac{1}{2}(300)(300) \sin 72$

$K = 42,797.54323$

$\times 5$

$K = 213,987.7162$

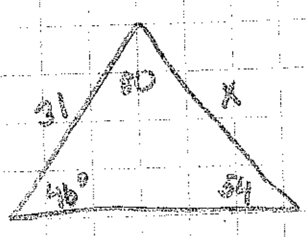
$\boxed{K = 213,988 \text{ ft}^2}$

What if we are not given 2 sides and an included angle?

<u>Option 1</u>: Use Law of Sines / Cosines to find whats missing

<u>Option 2</u>: Memorize 2 more formulas!

Find the area to the nearest square meter:



$\frac{x}{\sin 46} = \frac{31}{\sin 54}$

$\frac{\sin 54 \cdot x}{\sin 54} = \frac{31 \cdot \sin 46}{\sin 54}$

$x = 27.56373966$

$K = \frac{1}{2} ab \sin C$

$K = \frac{1}{2}(31)(27.56373966) \sin 80$

$K = 420.74726$

$\boxed{K = 421 \text{ m}^2}$

The following formulas are used to find the area of any triangle... you only need to know the first one!

2 sides & 1 angle: $K = \frac{1}{2}ab\sin C$

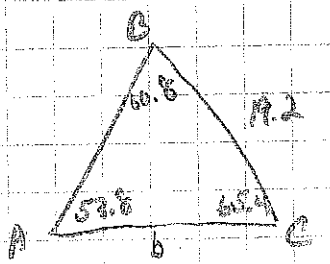
1 side & 2 angles: $K = \frac{1}{2}a^2\frac{\sin B\sin C}{\sin A}$

3 sides: $K = \sqrt{s(s-a)(s-b)(s-c)}$

where s = semi perimeter $= \frac{1}{2}(a+b+c)$

Examples

1) In $\triangle ABC$, $a = 19.2$, $A = 53.8°$, and $C = 65.4°$
Find the area to the nearest tenth



$$\frac{b}{\sin 60.8} = \frac{19.2}{\sin 53.8}$$

$$\frac{\sin 53.8 \cdot b}{\sin 53.8} = \frac{19.2 \cdot \sin 60.8}{\sin 53.8}$$

$b = 20.76942773$

$K = \frac{1}{2}ab\sin C$

$K = \frac{1}{2}(19.2)(20.76942773)\sin 65.4$

$K = 181.2894111$

$\boxed{K = 181.3 \text{ units}^2}$

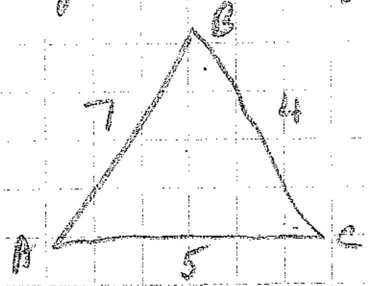
OR

$$K = \frac{1}{2}a^2\frac{\sin B\sin C}{\sin A}$$

$$K = \frac{1}{2}(19.2)^2\frac{\sin 60.8 \cdot \sin 65.4}{\sin 53.8}$$

$K = 181.2894111$ $\boxed{K = 181.3 \text{ units}^2}$

2) In ΔABC, $a=4$, $b=5$, and $c=7$. Find the area of the triangle to the nearest hundredth.



$S = \frac{1}{2}(a+b+c)$

$S = \frac{1}{2}(4+7+5)$

$S = \frac{1}{2}(16)$

$S = 8$

$K = \sqrt{s(s-a)(s-b)(s-c)}$

$K = \sqrt{8(8-4)(8-7)(8-5)}$

$K = \sqrt{8(4)(1)(3)}$

$K = \sqrt{96}$

$K = 9.797958971$

$\boxed{K = 9.80 \text{ units}^2}$

Homework:

1) In ΔBUG, $b=11.5$, $u=13.7$ $g=12.2$. Find the area.

2) In ΔBAT, $b=5$, $A=37°$, and $T=84°$. Find the area.

3) Find the area of an octagon inscribed in a circle that has a radius of 5.